# Performance Analysis of the Probabilistic Multi-Hypothesis Tracking Algorithm On the SEABAR Data Sets

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Abstract- The Probabilistic Multi-hypothesis Tracking (PMHT) algorithm [1] is a batch type multi-target tracking algorithm based on the Expectation-Maximization (EM) method [2]. Unlike other more popular methods (e.g., Multi-Hypothesis Tracking, MHT) the computational burden of PMHT grows linearly in the size of the batch, the number of clutter detections, and the number of targets tracked.

The SEABAR [3] sea tiail was conducted by the NATO Undersea Research Center in 2007 to investigate the suitability of some experimental high gain deployed active sonar receivers for tracking underwater contacts of interest. The sea trial yielded several useful multi-static active sonar data sets. The purpose of the effort reported here is to assess the target tracking performance of PMHT using structured multi-static active sonar sea trial data collected during the SEABAR experiment. This study quantifies the effects of batch size on the ability of PMHT to hold track on constant velocity and maneuvering contacts to determine the values that provide acceptable tracking performance. Situations involving contact maneuvers or temporary loss of detection (a.k.a., drop outs) are of particular interest. Specifically, the ability of PMHT to hold track as a function of batch size for two multi-static active sonar sea trial data sets containing contact maneuvers and drop outs will be assessed.

**Keywords**- Probabilistic Multi-hypothesis Tracker, multi-static active sonar, batch target tracking, batch size, centralized and distributed processing systems.

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### 1 Introduction

Batch tracking algorithms (e.g., Multi-Hypothesis Tracking, MHT) are currently being developed and investigated for multi-static active sonar systems to improve overall tracking performance in general and track hold during contact maneuvers and temporary loss of detection in particular. The cost of estimating a sequence of target states instead of just the current state (i.e., recursive tracking) is increased computational burden and for many batch algorithms the computational load increases factorially with the length of the batch. The Probabilistic Multi-Hypothesis Tracking (PMHT) algorithm considered here enjoys the advantage of having a computational load that grows linearly in the length of the batch, the number of contacts and the density of clutter. This is true even if the targets tracked are in close proximity or crossing.

The reason many batch methods exhibit combinatorial growth in their computational burden is because they are based on the assumption of at most one detection per target per scan per sensor which requires the enumeration of every possible sequence of detection to track assignments to determine the optimal estimate of target trajectory. By contrast PMHT is based on a more flexible assignment model that allows for the possibility of more than one target detection. Many schemes have been developed to control the computational burden of MHT type methods by limiting the selection of assignment sequences to the most likely, [4]. Such pruning methods, however, inevitably sacrifice any optimality property of the full algorithm. The computational burden of any algorithm increases with batch size and therefore system designers will need to know the

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14 ABSTRACT

The Probabilistic Multi-hypothesis Tracking (PMHT) algorithm [1] is a batch type multi-target tracking algorithm based on the Expectation-Maximization (EM) method [2]. Unlike other more popular methods (e.g. Multi-Hypothesis Tracking, MHT) the computational burden of PMHT grows linearly in the size of the batch, the number of clutter detections, and the number of targets tracked. The SEABAR [3] sea tiail was conducted by the NATO Undersea Research Center in 2007 to investigate the suitability of some experimental high gain deployed active sonar receivers for tracking underwater contacts of interest. The sea trial yielded several useful multi-static active sonar data sets. The purpose of the effort reported here is to assess the target tracking performance of PMHT using structured multi-static active sonar sea trial data collected during the SEABAR experiment. This study quantifies the effects of batch size on the ability of PMHT to hold track on constant velocity and maneuvering contacts to determine the values that provide acceptable tracking performance. Situations involving contact maneuvers or temporary loss of detection (a.k.a., drop outs) are of particular interest. Specifically, the ability of PMHT to hold track as a function of batch size for two multi-static active sonar sea trial data sets containing contact maneuvers and drop outs will be assessed.

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minimum batch size required to achieve the desired performance.

The analysis presented here utilizes a centralized processing architecture where the measurements (i.e., clustered echo detections) are registered to a common frame of reference and synchronized. Figure 1 illustrates the fundamental cycle of a centralized tracking architecture that performs sequential updates to a set of tracks using registered synchronized measurements from two sensors.

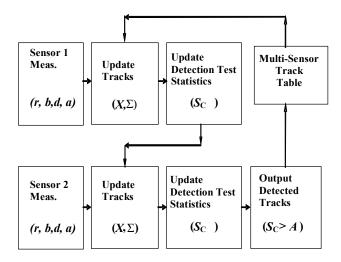


Figure 1. Centralized Multi-Sensor Tracking and Detection

In this analysis an implementation of PMHT based on the centralized architecture depicted in figure 1 is used to evaluate track hold performance as a function of sensor registration error and target SNR. The PMHT algorithm is an adaptation of the Expectation-Maximization (EM) method [2] that is formulated to estimate batch state sequences of multiple maneuvering targets in clutter from active or passive sonar detections. PMHT is explicitly designed to utilize data that is fundamentally incomplete; the measurements contain no information on their origin (e.g., target or clutter). The derivation of the original PMHT algorithm is well described in [1] and is based on the so called independent assignment model; each measurement has some non-zero prior probability of being from any one of the targets present independent of the origin of all the other measurements. Under this assignment model it is entirely possible for all of the measurements to originate from any one of the targets but that hypothesis is almost always far less likely than more sensible assignments.

The advantage of the independent assignment model is that when it is used in conjunction with the Expectation Maximization method it avoids having to enumerate a large number of candidate measurement assignment hypotheses and instead only requires the calculation of the posterior probabilities that the *r'th* measurement at time *t* originated from target *s* as

$$w_{str} = \frac{\pi_s N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\sum_{m=1}^{M} \pi_m N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})}$$
(1)

where  $\pi_i$  is the prior probability that a measurement originated from the *i'th* target being tracked and M is number of targets. In [1] and [4] the above formula is modified to employ amplitude information and account for uniformly distributed clutter

$$w_{str} = \frac{\pi_s f_1(a_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{ts}, \mathbf{R}_{ts})}{\frac{\pi_0}{V} f_0(a_{rt}) + \sum_{m=1}^{M} \pi_m f_1(a_{rt}) N(\mathbf{z}_{rt}; \mathbf{x}_{tm}, \mathbf{R}_{tm})}, (2)$$

where V is the volume of the association gate, and  $f_0(a_{rt})$  and  $f_1(a_{rt})$  are the distributions for the echo amplitudes for clutter and target respectively. In this study the thresholded target echo amplitudes are assumed to be Rayleigh distributed;

$$f_1(a) = \frac{\pi a}{2 P_D(1+\theta)} e^{-\pi a^2/4(1+\theta)} \text{ for } a > \tau.$$
 (3)

and the clutter distribution is a thresholded unit mean Rayleigh;

$$f_0(a) = \frac{\pi a}{2P_{fa}} e^{-\pi a^2/4}, \text{ for } a > \tau.$$
 (4)

The basic procedure of the modified PMHT algorithm used in this study amounts to iterating the following four steps:

- Compute the association weights, w str , for each measurement and target at each time step in batch.
- 2. Using the weights compute a measurement centroid and effective error covariance matrix (a.k.a. the synthetic measurement and covariance) for each target at each time step in the batch.
- 3. Update the track (i.e., the batch sequence of state estimates) for each target with a Kalman smoother on the synthetic measurements and error covariance matrices.
- 4. Compute the true error covariance of the measurement centroids from step 2. Compute the error covariance of the state estimates using the true measurement error covariance and the Kalman gains from step 3.

## 2 Purpose

The purpose of the effort reported here is to assess the target tracking performance of PMHT using structured multi-static active sonar sea trial data collected during the SEABAR [2] experiment. This study quantifies the effects of batch size on the ability of PMHT to hold track on

constant velocity and maneuvering contacts to determine the values that provide acceptable tracking performance. Situations involving contact maneuvers or temporary loss of detection (a.k.a., drop outs) are of particular interest. Specifically, the ability of PMHT to hold track as a function of batch size for two multi-static active sonar sea trail data sets containing contact maneuvers and drop outs will be assessed.

In order to focus on the effect of batch length the tracking conditions are assumed to be ideal in most other respects: independent and identically distributed zero mean measurement errors with known covariance and a benign environment with identical interference level, propagation loss and target strength at all sensors. An important exception is that the distributions of the spatial density and amplitudes of the clutter detections are not assumed to be the same for both FM and CW waveforms. The FM and CW data used in this study contains a significant amount of clutter that exhibited markedly different spatial densities and amplitude distributions. The reconstruction information was used to separate the clutter detections from the target detections for each waveform and estimate the spatial density and amplitude distribution for use in the PMHT tracker. This was necessary because the amplitudes of the clutter data were not well modeled by standard ideal normalizer output (i.e., unit mean Rayleigh).

In this way the values of batch size that provide acceptable track hold will be identified allowing multi-static systems engineers to determine the suitability of such a design for various applications.

### 3 The SEABAR Data

The SEABAR experiment conducted in the Mediterranean Sea in 2007 collected numerous structured data sets from various multi-static active and passive sonar systems. The data sets used in this analysis involved a fixed position active source that transmitted FM and CW waveforms and several fixed active sonar buoy receivers. A single target of interest was simulated using a towed echo repeater along a trajectory designed to simulate realistic low and high Doppler scenarios.

Three data sets from events A01, A05, and A06 are use in this analysis. In each set the target detections have been corrected for irregular echo repeater delay and the observed amplitudes have been replaced with values calculated with the aspect and range dependent BASIS target model. The data from event A05 and A06 were concatenated into one data set subsequently denoted A56.

Figure 2 is a histogram of the FM clutter detection SNRs (i.e., amplitudes) for data set A01 and it clearly shows a lower threshold of 6.5 dB. This data is highly non-Rayleigh and was instead modeled by a two component Gauuian mixture distribution.

Figure 3 is the histogram of the BASIS model injected FM target detection SNRs for data set A01. Many of the injected SNR values are below the apparent detection threshold for the FM waveform and all such detections were

omitted in this analysis. This effectively lowered the probability of FM target detection by about 30%.

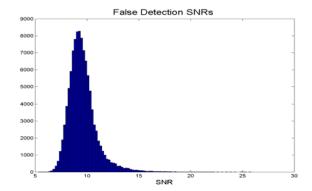


Figure 2. Histogram of False FM Detection SNRs in the A01 SEABAR Data Set.

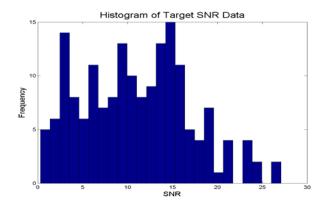


Figure 3. Histogram of the Injected FM Target Detection SNRs in the A01 SEABAR Data Set.

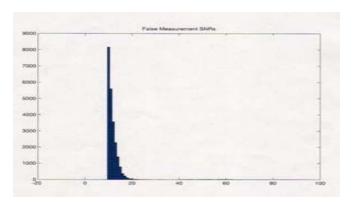


Figure 4. Histogram of False CW Detection SNRs in the A01 SEABAR Data Set.

The situation for the CW waveform is similar. Figure 4 is a histogram of the CW clutter detection SNRs for data set A01 and it clearly shows a lower threshold of 10 dB. Figure 5 is the histogram of the CW injected target detections for data set A01. As with the FM data, many of the injected CW SNR values are below the apparent detection threshold of 10 dB and all such detections were omitted in this analysis. This effectively lowered the probability of CW target detection by about 40%. The amplitudes of the remaining true target FM and CW detections were modeled with a Rayleigh distribution matched to a target exhibiting an average SNR of 15 dB.

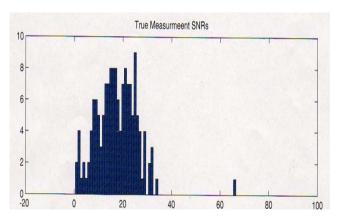


Figure 5. Histogram of the Injected CW Target Detection SNRs in the A01 SEABAR Data Set.

The effect of thresholding the injected target SNRs is even more severe for the A56 data set; approximately 40% of FM detections are lost and about 75% of CW detections are lost. For the A56 data set the assumed average target SNR was reduced to 10dB.

The data is each test set were registered to common frame of reference before tracking to investigate a centralized tracking architecture. All target tracks were initialized by a method based on Hough bin partitioning of the data and Page's test [5]. Target tracks were rapidly initialized in both data sets; the initialization latency was 5 pings or less. The performance metric considered here is essentially track hold. Other popular metrics (e.g., probability of detection, false alarm rate, track detection latency) could not be easily evaluated due to the limited number of true targets and the nature of the statistics of the injected target amplitudes.

The implementation of PMHT applied here incorporates a piecewise linear white noise acceleration model with  $\sigma = 0.01 \text{yd/sec}^2$  and a Gaussian measurement error model having range error standard deviation of 150 meters and bearing error standard deviation of 2.0 degrees. The data association gates in this implementation of PMHT were computed from the state error covariance provided by the modified PMHT calculations (i.e., step 4 in Section 1) and designed to have a 99.9% probability of target containment.

### 4 Results

The A01 event consisted of a single source and three buoyed receivers as shown in Figure 6. The source transmitted FM and CW waveforms and the positions of the source and receivers were maintained during the event.

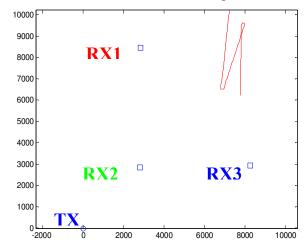


Figure 6. SEABAR event A01 geometry. Transmitter is at the origin and the three buoyed receivers are located as shown. The planned run for the towed echo repeater is shown in red and consists of three legs with a "Crazy Ivan" maneuver between the second and third leg.

Figures 7 and 8 show the target tracking performance of PMHT on the thresholded A01 FM and CW data separately using a batch size of 5 pings. The batch length of 5 pings was chosen to match that used by other researchers investigating other batch algorithms. In each case the data clearly exhibits bias between the sensors (i.e., the data from different sensors are not perfectly registered) drop outs in target detection during the first maneuver and after the second maneuver. In both cases PMHT is unable to hold track through the entire event.

PMHT can be formulated to process both CW and FM measurements to estimate the state sequence. Figure 9 shows the target track estimated by PMHT using FM and CW SEABAR data. PMHT clearly holds track through the first leg and does not get drawn off by clutter during the target dropout occurring at the first maneuver. PMHT resumes tracking the target after the dropout and holds track through the "Crazy Ivan" maneuver at the end of the second leg. These results show that using the FM and CW data from this event in a single PMHT significantly improved tracking performance. In each case the performance did not significantly improve with longer batch lengths; the only configuration of PMHT that could hold track through the entire data set used both FM and CW measurements.

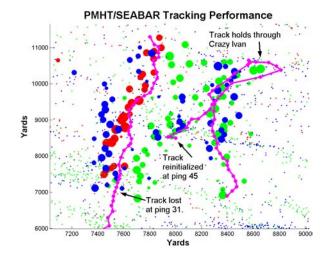


Figure 7. PMHT Tracking performance on SEABAR event A01 FM data with 5 ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. Red dots are detections from receiver1, green dots receiver 2, and blue dots receiver 3. PMHT is unable to hold track through drop out at first maneuver (i.e., end of first leg) but hold through "Crazy Ivan" maneuver after track is reinitialized.

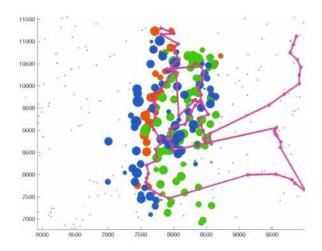


Figure 8. PMHT Tracking performance on SEABAR event A01 CW data with 5 ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. Red dots are detections from receiver1, green dots receiver 2, and blue dots receiver 3. PMHT is unable to hold track through drop out at first maneuver (i.e., end of first leg) but reacquires target and holds through "Crazy Ivan" and then loses track again.

However, when combining FM and CW measurements into one data stream the batch size could significantly be reduced and still obtain acceptable track hold performance. Figure 10 shows the target track estimated by PMHT using FM and CW data and a batch size of two pings (i.e., 4 individual scans of data; 2 FM and 2 CW). As the next data set will show, these results cannot be obtained in every case but they do illustrate the performance improvement that can

be obtained by combining FM and CW measurements when both waveforms are detecting the target.

The A56 event consisted of a single source and two buoyed receivers as shown in Figure 11. The source transmitted FM and CW waveforms and the positions of the source and receivers were maintained during the event.

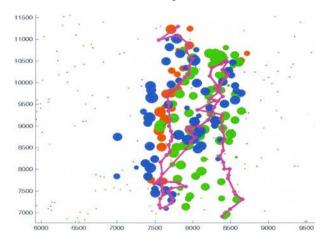


Figure 9. PMHT Tracking performance on SEABAR event A01 using FM and CW data with 5 ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. Red dots are detections from receiver1, green dots receiver 2, and blue dots receiver 3. PMHT holds track through entire event.

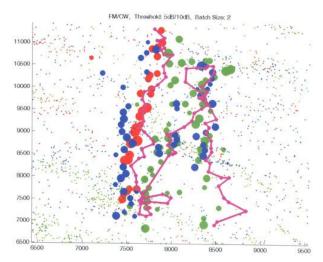


Figure 10. PMHT Tracking performance on SEABAR event A01 using FM and CW data with two ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. Red dots are detections from receiver1, green dots receiver 2, and blue dots receiver 3. PMHT holds track through entire event.

Figures 12 and 13 show the target tracking performance of PMHT on the A56 event FM and CW data separately using batch sizes of 12 and 20 pings respectively. PMHT is able to hold track through the entire run using FM data alone. The track quality during the first maneuver and the drop out at the end of the first leg, however, are clearly minimal and barely adequate to hold track. PMHT was unable to hold track for smaller batch sizes using solely the

FM data and tracking performance did not significantly improve with larger batch lengths. The probability of target detection with the CW waveform is about 10% after thresholding and consequently PMHT is unable to hold track through the first maneuver using CW data alone as shown in figure 13. The same performance was achieved with other batch lengths.

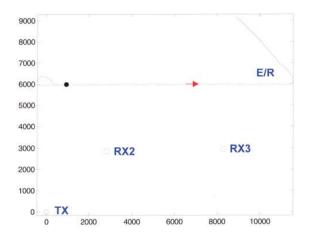


Figure 11. SEABAR event A56 geometry. Transmitter is at the origin and the two buoyed receivers are located as shown, receiver 1 was unavailable for this event. The planned run for the towed echo repeater is shown in red and consists of two legs beginning with a "Crazy Ivan" maneuver.

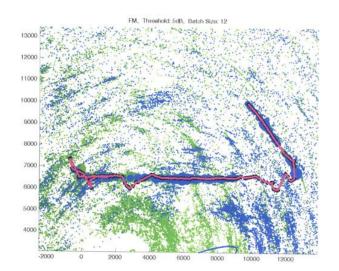


Figure 12. PMHT Tracking performance on SEABAR event A56 FM data thresholded at 5dB using a with twelve ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. PMHT holds track through drop out at end of first leg.

Figure 14 shows the target track estimated by PMHT using A56 event FM and CW SEABAR data in the same track. PMHT clearly holds track through the first maneuver and does not get drawn off by clutter during the target dropout occurring at the end of the first leg. PMHT

resumes tracking the target after the dropout and holds track through the end of the second leg.

These results show that using the FM and CW data from this event in a single PMHT significantly improved tracking performance during the first leg and modestly overall. However, using combined waveforms did not afford the ability to significantly decrease the batch length and maintain acceptable performance on this data set.

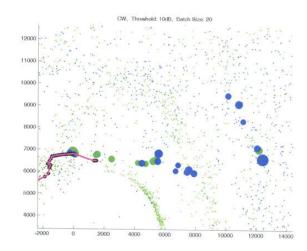


Figure 13. PMHT Tracking performance on SEABAR event A56 CW data thresholded at 10dB with 20 ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. PMHT is unable to hold track through initial maneuver at the beginning of the first leg. Target is never reacquired.

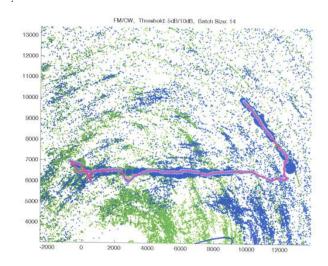


Figure 14. PMHT Tracking performance on SEABAR event A56 using FM and CW thresholded data with 14 ping batch. Target detections are plotted in large dots and clutter detections are plotted in small dots. PMHT holds track through entire event.

#### 5 Conclusions

The results in the preceding section clearly show that PMHT can achieve exceptional track hold performance with small to moderate batch lengths. Moreover, the track hold performance was robust to target maneuvers, temporary loss of target detection, aspect dependent target SNR, and moderate registration error. Although the batch size had to be increased for the lower target SNR data set (i.e., A56) it should be noted that the extended drop out was caused by a malfunction in the towed echo repeater. A genuine target would almost certainly have provided much better detections during this interval and required a smaller batch size to hold track.

The results presented here clearly show that PMHT provides at least competitive, and possibly impressive, multi-static tracking performance on active sonar sea trial data. Moreover, PMHT offers computational efficiency and system flexibility; it can be implemented in either distributed or centralized architectures and combined with almost any track management logic. Appropriate track initialization methods have been presented in [8]. PMHT is a viable multi-target tracking method and it should be considered for use in fielded multi-static active sonar systems.

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